Analysis of technical and economical data
Tutorial problem series for the 2nd midterm, 2019 fall

1. Supposing $E(\xi) = 3$ and $E(\xi^2) = 11$, determine the expected value $E(\eta)$ if $\eta = (4\xi + 1)^2$!
   (solution: $E(\eta) = 201$)

2. Considering the properties of the probability density function, determine the value of the parameter a occurring in the probability density function $f(x)$ of the random variable ξ, then determine its expected value $E(\xi)$! $(solution\colon a = 3,\ E(\xi) = 3/4)$
   $$\xi \in \left(0, \frac{a}{3}\right), \qquad f(x) = ax^2$$

3. Let the probability density function be:
   $$f(x) = \begin{cases} \frac{3}{4}x(2-x) & if\ 0 \le x \le 2 \\ 0 & otherwise \end{cases}$$
   Determine the expected value and the standard deviation of ξ!
   $(solution\colon E(\xi) = 1,\ Var(\xi) = \frac{1}{5}\ is\ the\ variance,\ \sqrt{Var(\xi)} = \frac{1}{\sqrt{5}}\ is\ the\ standard\ deviation\ )$

4. The probability density function of a continuous random variable ξ is:
   $$f(x) = \begin{cases} \frac{4}{5}(1+x) & if\ -1 < x < 0 \\ \frac{4}{15}(3-2x) & if\ 0 \le x < \frac{3}{2} \end{cases}$$
   Derive the distribution function of ξ!
   (Solution:
   $$F(x) = \begin{cases} 0, & if\ x \le -1 \\ \frac{2}{5}(x+1)^2, & if\ -1 < x < 0 \\ 1 - \frac{1}{15}(2x-3)^2, & if\ 0 \le x < \frac{3}{2} \\ 1, & if\ x \ge \frac{3}{2} \end{cases}$$
   Little help: F(x) in 3/2 should converge to 1, and in -1 to 0!)

5. The probability density function of a continuous random variable ξ is:
   $$f(x) = \begin{cases} \frac{2}{9}(x-1) & if\ 1 \le x \le 4 \\ 0 & otherwise \end{cases}$$
   Determine the probability distribution function $F(x)$ and calculate the expected value $E(\xi)$ of ξ! Try to find the median of the distribution of this random variable too!

   (Solution:

$$F(x) = \begin{cases} 0, & if\ x \leq 1 \\ \frac{1}{9}(x-1)^2, & if\ 1 \leq x \leq 4 \\ 1, & if\ x > 4 \end{cases}$$

$E(\xi) = 3$. The median: $F^{-1}(1/2) = 1 + 3\sqrt{\frac{1}{2}} = 3.121$.)

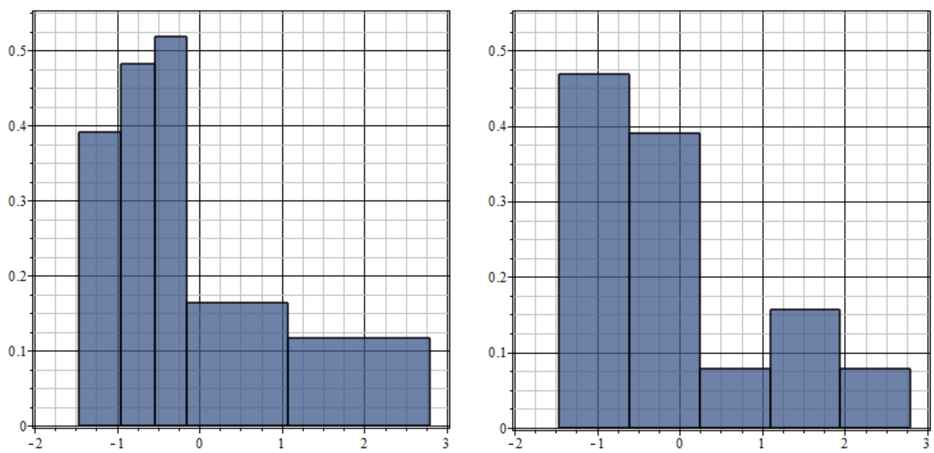
6. Describe the steps of the construction of the empirical density function for an outcome of n elements!

7. Sketch a box-plot! Name and define the quantities used for making the box-plot!

8. The manufacturer of mobile phones makes a research in order to find out how much money the interviewed people have spent for their phones (in thousand forints). The results are included in the right hand column of the table.
    a. Prepare the empirical density function of the expenses!
    b. Explain the results!

Serial number	**Price (1000 FT)**
1	4
2	6
3	10
4	12
5	12
6	23
7	24
8	38
9	40
10	51
11	55
12	60
13	60
14	87
15	90
16	120

9. The two histograms below have been prepared based on 15 elements of a sample with normal distribution N(0,1). In the first case the learnt method has been applied while in the second one the sample volume was divided into equal intervals. Find the frequency of the interval [-



0.5, 1.5] (very high accuracy is not required) using both histograms! Based on the shapes of the histograms declare which of them represents the data better and why?

10. What kind of quantity proves linear relation between two random variables? Give the formula, explain the notations and interpret the meaning of the result of the formula!

11. Give the empirical correlation coefficient, define the notations! On what does this quantity inform us and how?

12. One possible cause of the corrosion of concrete drain-pipes is sulphuric acid being produced by bacteria in sulphuric surroundings. The speed of corrosion must be known for lifetime estimation of these pipes. The table below contains the thicknesses (d) of corroded material for pipes of different ages (t). Both parameters are loaded with errors (exact dates of tests are missing, only the years were saved and the corroded area is not equal around the perimeter of pipes). Fit data with a straight line using a valid method! ($Wald\ method, d = at + b, a = 4{,}21,$ $b = -0{,}32$)

t (year)	1	5	2	7	3	8
d (mm)	4,1	21,2	8,4	30,1	11,8	31,9

13. Fit the points given in the table by a function $f(x) = ax^2$ using an appropriate method (Independent variable values x are not loaded with errors). (Least squares method, $a = 3{,}59$)
Determine the goodness of fit! ($R^2 = 0{,}999773$)

x	**y**
2	16
4	60
9	290

14. A measuring orifice is used to measure the flow rate *(Q)* of a fluid flowing in a circular tube. The mathematical relationship is shown below. Determine the relative standard deviation of the calculated flow rate, if the diameter *(d)* is 100 ±1mm, the pressure difference *(Δp)* is 1000 ± 5 Pa, the density *(ρ)* is 1.2 ± 0.1 kg / m³ and the other parameters *(α, ε)* are constant. The error limit is twice the standard deviation.

$$Q = \alpha\varepsilon \frac{d^2\pi}{4}\sqrt{\frac{2\Delta p}{\rho}}$$

$$h_Q = \frac{\sigma_Q}{Q} =$$

15. The velocity of a passing train has to be determined. We measure the time (4,2 s) and the distance (200 m). The standard deviation of the time measurement is 0,15 seconds, the standard deviation of the distance measurement is 2,25 meters. Calculate the velocity, the standard deviation of the velocity and the relative standard deviation (or relative variance) of the velocity!